Mathematical Modeling Of Multi-Units Pumping Station With Asynchronous Electric Drive Of Centrifugal Pumps

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Abstract. *Based on the system approach and the principle of electrohydrodynamic analogy, a formalized mathematical model of a generalized system of asynchronous centrifugal pumping units of a multi-unit pumping station with an asynchronous electric drive of centrifugal pumps was created. This model is easy to adapt to the specific configuration of the pumping station. A feature of the model is the use of such parameters of the centrifugal pump, which are calculated directly through the geometric dimensions of its internal elements, taking into account the dependence of these parameters on the physical properties of the working fluid. Unlike existing models, this makes it possible to take into account the influence of operational or emergency changes in pump parameters and operational changes in the physical properties of the working fluid both on its pumping regimes and on the operation regimes of the electric drive directly during the simulation. The scope of use and ways of improving the developed model, as well as the direction of further research, are proposed.*

Keywords

Induction machine, centrifugal pump, model, pipeline, pumping station.

1. Introduction

Transportation of large volumes of liquid to distribution pipelines (PL) provides transit objects - main PL and pump stations (PS) that consume significant volumes of electricity and have a strategic value. A high-performance coefficient of modern centrifugal pumps (CP) makes a tangible potential of energy efficiency in non-stationary regimes. Powerful PS of the main oil pipelines or water pipelines is complex technological complexes, whose subsystems are inextricably combined by a single technological process and function as single systems. The requirements for uninterruptedly, reliability, the safety of operation, fault finding, and diagnosis of existing, as well as requirements for the design of new such strategic objects are usually regulated by national legislation (for example [1] in Ukraine). The possibility of physical experiments to analyze the regimes of operation of such complexes is substantially limited by the inadmissibility of interruption of their functioning, as well as technical and financial risks. The use of a hybrid model consisting of a digital model of power equipment and a physical model of management system, allows to solve this problem [2]. Thus, the creation of an effective tool for computer research of powerful power equipment of pumping stations to avoid physical experiments on them as much as possible is an urgent task.

Most of the works, which describe the study of the regimes of operation of the main PS power equipment by modeling, can be divided into four groups. We will show it in an example of some publications. The first of them is characterized by an in-depth mathematical description of only the hydraulic [3], only electromechanical [4], or only control [5] subsystem of the PS or single asynchronous centrifugal pump unit (ACPU), the other subsystem is presented very simply. The same group includes work on the diagnosis of faults in the hydraulic subsystem of the PS based on the analysis of the behavior of its electromagnetic subsystem [6]. The use of such an approach is appropriate for solving of limited amount of specific problems. The second group of works can include imitation models [7], which are useful for calculating the basic coordinates of operating regimes, but do not reflect the physical nature of processes in any of the PS subsystems. The third group of works is devoted to the optimization of general energy indicators of operating regimes of individual PS and PS cascades [8]. The comparable degree of detail of the mathematical description of the inextricably combined hydraulic and electromechanical subsystems of PS characterizes the fourth group ([9] and [10]). This makes it possible to study the regimes of the PS, taking into account the mutual influence of subsystems of different physical natures and the parameters of their internal elements. However, the functionality of the model [9] is limited by the ability to study the dynamic regimes of only a single-unit PS, and the functionality of the model [10] of multi-unit PS is limited by the ability to study only steady-state regimes. Thus, the analysis of works devoted to this topic shows the absence of an effective multifunctional tool for computer research of such objects. This work aims to develop a generalized mathematical model for studying the dynamic regimes of operation of an arbitrary system of asynchronous centrifugal pump units of a multi-unit pumping station with an asynchronous electric drive of centrifugal pumps.

2. Mathematical Model of Multi-Units Pumping Station

The connection scheme of the hydraulic subsystem ACPU generalized multi-unit PS is shown in Fig. 1. The connection scheme of the hydraulic subsystem ACPU of the generalized multi-unit PS is shown in Fig. 1. This scheme shows N_{PS} units with a combined connection hydraulic tracts of CP. Units whose pumps are connected in series are grouped into N groups. Hydraulic tracts of units of all groups are connected in parallel with one common pipeline. Each of the groups of units, the hydraulic tracts of which are connected in parallel, includes a different number of separate units, the hydraulic tracts of the pumps which are connected in series. The symbol M_j indicates the total number of pumps connected in series by hydraulic tracts in the j-th group ($j = 1..N$ - serial number of the group). For numbering of the i-th separate ACPU, IM, CP in the j-th group the notation i, j is used. The total number of PS units is:

$$N_{PS} = \sum_{j=1}^{N} M_j, \tag{1}$$

The equations of the mathematical model CP are written based on the principle of electrohydrodynamic analogy in the system of rotating orthogonal d-q coordinates, rigidly connected by the CP impeller (based on the developments given in [11] and [12]). In this coordinate system, the equivalent hydraulic resistances and inductivities of CP are of constant value, and the pressures and flow rates of the working fluid are harmonic functions of time, which allows the use of the apparatus of a complex variable. The actual instantaneous values of pressures and flow rates are calculated as modules of the corresponding complex values. The parameters of the CP are identified by the geometric dimensions of its internal elements, taking into account the influence on the parameters of kinematic viscosity and density of the working fluid. The equations of the mathematical model of a saturated induction machine (IM) are formed in the same coordinate system [13]. Saturation is taken into account by using the polynomial approximation of the static nonlinear inductive elastance of the main magnetic circuit IM. The motor and the pump are connected by a completely rigid shaft. PL is presented in the form of static counter-pressure, equivalent to hydraulic resistances and inductivities. A per-unit system is used in all equations and expressions; exceptions are the total moment of inertia ACPU $J_{\sum}$ in (kg.m2), time t in (s), as well as the basic values [9], denoted by the symbol "b".

The mathematical model of the ACPU system of the generalized multi-unit PS with the combined combination of CP hydraulic tracts is represented by equations (2)...(10). In these equations, the dot above the symbols denotes complex variables (coordinates d-q corresponds to real and imaginary parts):

$$\frac{\mathrm{d}\mathbf{Q}_{i,j}}{\mathrm{d}t} = \mathbf{P}_{i,j} \times \mathbf{Q}_{i,j} + \mathbf{P}_{0_{i,j}} \cdot \dot{h}_{0_{i,j}} + \mathbf{V}_{i,j}, \tag{2}$$

$$h_{CPd_{i,j}} q_{33q_{i,j}} - h_{CPq_{i,j}} q_{33d_{i,j}} = 0, \tag{3}$$

$$\dot{h}_{0_{i,j}} = H_{0nom_{i,j}} \left(\frac{\omega_b}{\omega_{CPb_{i,j}}} \right) \omega^2_{r_{i,j}} e^{j\omega_{CPb_{i,j}} \omega_{r_{i,j}} t}, \tag{4}$$

$$\begin{aligned} \frac{\mathrm{d}\omega_{r_{i,j}}}{\mathrm{d}t} &= \frac{1}{J_{\Sigma_{i,j}} \omega_{IMb_{i,j}}} \cdot \Big(T_{IMb_{i,j}} \big(\psi_{\delta d_{i,j}} i_{sq_{i,j}} - \\ &- \psi_{\delta q_{i,j}} i_{sd_{i,j}}\big) - T_{CPb_{i,j}} H_{0nom_{i,j}} \frac{\omega_{IMb_{i,j}}}{\omega_{CPb_{i,j}}} \omega_{r_{i,j}} \\ &\times \sqrt{(q_{11d_{i,j}} + q_{44d_{i,j}})^2 + (q_{11q_{i,j}} + q_{44q_{i,j}})^2} \Big), \end{aligned} \tag{5}$$

$$\left(\frac{Q_{CPb_{i,j}}}{Q_b} \right)^2 \left(q^2_{33d_{i,j}} + q^2_{33q_{i,j}} \right) = q^2_{33d_{i,j+1}} + q^2_{33q_{i,j+1}}, \tag{6}$$

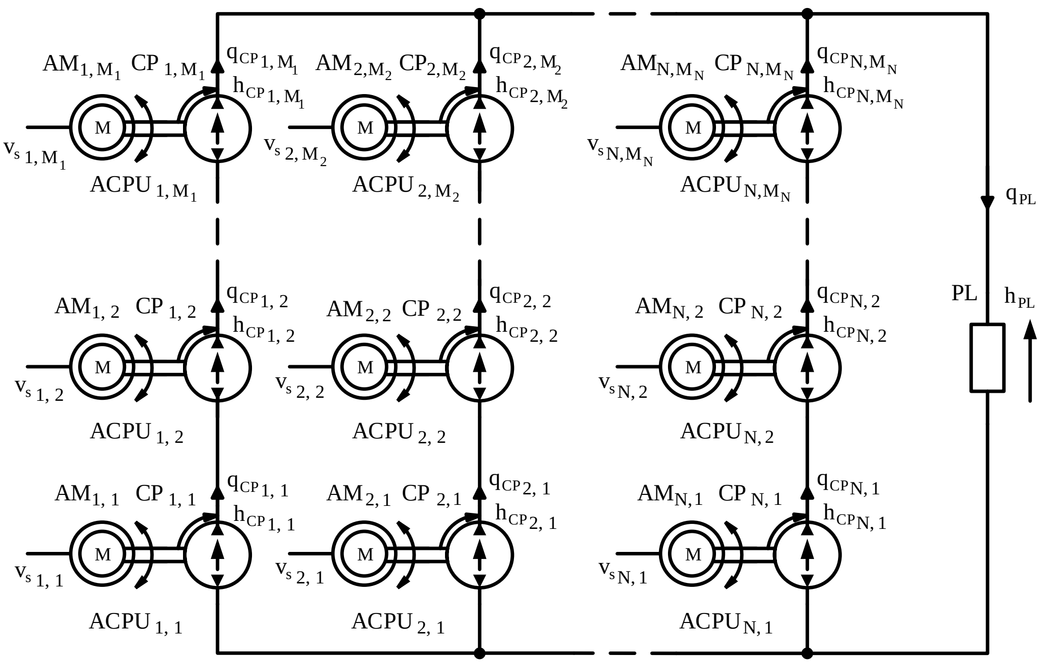


Fig. 1: The connection scheme of the hydraulic subsystem ACPU generalized multi-unit PS.

$$\left(\frac{H_{CPb_{i,j}}}{H_b}\right)^2\left(h^2_{CPd_{i,j}}+h^2_{CPq_{i,j}}\right)$$
$$=\left(\frac{H_{CPb_{i,j}}}{H_b}\right)^2\left(h^2_{CPd_{i,j+1}}+h^2_{CPq_{i,j+1}}\right), \tag{7}$$

$$q_{PL}=\sum_{j=1}^{N}\frac{Q_{CPb_{i,j}}}{Q_b}\sqrt{q^2_{33d_{i,j}}+q^2_{33q_{i,j}}}, \tag{8}$$

$$h_{PL}=\sum_{i=1}^{M_j}\frac{H_{CPb_{i,j}}}{H_b}\sqrt{h^2_{CPd_{i,j}}+h^2_{CPq_{i,j}}}, \tag{9}$$

$$\frac{\mathrm{d}q_{PL}}{\mathrm{d}t}=-\frac{r_{PL}}{L_{PL}}q_{PL}+\frac{1}{L_{PL}}h_{PL}-\frac{1}{L_{PL}}h_{st}. \tag{10}$$

where $\mathbf{P}_{i,j}$ is ACPU parameters (see Tab. 1 in detail), $\mathbf{Q}_{i,j}=\left(\dot{q}_{11_{i,j}},\dot{q}_{22_{i,j}},\dot{q}_{33_{i,j}},\dot{q}_{44_{i,j}},\dot{i}_{s_{i,j}},\dot{i}_{r_{i,j}},\dot{e}_{\delta_{i,j}},\dot{\psi}_{\delta_{i,j}}\right)_t$ is coordinates of the ACPU mode, $\mathbf{P}_{0_{i,j}}=\left(-L^{-1}_{11_{i,j}},-L^{-1}_{21_{i,j}},-L^{-1}_{31_{i,j}},L^{-1}_{m_{i,j}},0,0,0,0\right)_t$ is parameters of CP, $\mathbf{V}_{i,j}=\left(0,0,0,0,\dot{v}_{s_{i,j}}\cdot L^{-1}_{m_{i,j}},0,0,0\right)_t$ coercive forces of the ACPU, $\omega_{r_{i,j}}$angular speed of the common shaft ACPU, $\omega_{s_{i,j}}$ the angular frequency of the stator winding voltage, h_{st} is the PL static back pressure, $\dot{i}_{s_{i,j}}$ stator current IM, $\dot{i}_{r_{i,j}}$ rotor current IM, reduced to its stator, $\dot{v}_{s_{i,j}}$ stator winding voltage IM, $\dot{\psi}_{\delta_{i,j}}$ flux coupling from the magnetic flux of the air gap IM, reduced to its stator, $\dot{y}_{d_{i,j}}$ leakage inductance of the stator winding IM, $R_{s_{i,j}}$ resistance of the stator winding IM, $L_{r_{i,j}}$ leakage inductance of the rotor winding IM, reduced to its stator, $R_{r_{i,j}}$ the resistance of the rotor winding IM, reduced to its stator, $R_{\mu_{i,j}}\left(\psi_{\delta_{i,j}}\right)=\left(0.82+0.148\cdot\left(\psi^2_{\delta d_{i,j}}+\psi^2_{\delta q_{i,j}}\right)+0.044\times\left(\psi^2_{\delta d_{i,j}}+\psi^2_{\delta q_{i,j}}\right)^4\right)\cdot\left(x_{a_{i,j}}+x_{\sigma_{i,j}}\right)^{-1}$ is the static nonlinear magnetic reluctance of the IM main magnetic circuit, $x_{a_{i,j}}$ nominal magnetization inductance Induction, $x_{\sigma_{i,j}}$ nominal leakage inductance IM, $J_{\Sigma_{i,j}}$ is the total moment of inertia ACPU in (kg·m2), $T_{IMb_{i,j}}$ is the basic value of the electromagnetic torque AM in (N·m), $\omega_{IMb_{i,j}}$ is the basic value of angular speed of the IM in (s^{-1}), $T_{CPb_{i,j}}$ is the basic value of the mechanical torque CP in (N·m), $\omega_{CPb_{i,j}}$ is the basic value of angular speed of the CP in (s^{-1}), ω_b is the general base value of angular velocity in (s^{-1}), $\dot{h}_{0_{i,j}}$ estimated pressure of idealized CP, $H_{0nom_{i,j}}$ nominal pressure of idealized CP, $\dot{q}_{11_{i,j}}$, $\dot{q}_{22_{i,j}}$, $\dot{q}_{33_{i,j}}$, $\dot{q}_{44_{i,j}}$ are estimated fictitious volumetric flow rates CP, $h_{CP_{i,j}}=\sqrt{h^2_{CPd_{i,j+1}}+h^2_{CPq_{i,j+1}}}$ is the pressure at the CP outlet, $q_{CP_{i,j}}=\sqrt{q^2_{33d_{i,j}}+q^2_{33q_{i,j}}}$ volumetric flow rate at the CP outlet, h_{PL} pressure at the PL inlet, q_{PL} volumetric flow rate at the PL inlet, $r_{m_{i,j}}\left(q_{CP_{i,j}}\right)$ equivalent hydraulic resistance to take into account for the mechanical energy losses in the CP, L_{PL} hydraulic inductivity PL, r_{PL} hydraulic resistance PL.

Eq. (2), (3), (4), and (5) describe each individual (i-th) ACPU; the total number of Eq. is $11\cdot N_{PS}$, in particular:

- Eq. (2) is written in matrix-vector form; it describes the state of electromagnetic (4 equations with complex variables for each IM) and hydraulic (4 equations with complex variables for each SR) ACPU subsystems; $i = 1..M_j$, where $j = 1..N$; the total number of Eq. is $8 \cdot N_{PS}$.
- Eq. (3) describes the collinearity of the image vectors of pressure and volumetric flow rate of the working fluid at the output of each CP [11] (1 equation for each CP); $i = 1..M_j$, where $j = 1..N$; the total number of Eq. is N_{SP},
- Eq. (4) with a complex variable describes the dependence of the fictitious pressure CP on the rotational speed of their impellers; the total number of Eq. is N_{SP},
- Eq. (5) of the rotor motion IM describes the connection of the electromagnetic and hydraulic subsystems of each specific ACPU using a completely rigid common shaft; $i = 1..M_j$, where $j = 1..N$; the total number of Eq. is N_{SP}.

Eq. (6) and (7) describe the sequential combination of CP hydraulic tracts in each of the N ACPU groups and the parallel combination of all N ACPU groups; the total number of Eq. is N_{SP}, in particular:

- Eq. (6) describes the equality of volumetric flow rates of the working fluid of all series-connected hydraulic tracts CP j-th group (1 equation for the i-th and $i+1$-st CP of j-th group); $i = 1..M_j - 1$, where $j = 1..N$; the total number of Eq. is $N_{PS} - N$,
- Eq. (7) describes the equality of working fluid pressures of all N parallel connected groups (1 equation for j and $j+1$ group); $j = 1..N-1$; the total number of Eq. is $N_{PS} - N$.

Eq. (8), and (9) describe the parallel coupling of all N groups of ACPU and PL; the total number of Eq. is 2, in particular:

- Eq. (8) describes the equality of the volumetric flow rate of the working fluid at the input PL and the sum of the volumetric flow rates of the working fluid of all N groups of CP; $j = 1..N$; i is arbitrary in the range from 1 to M_j; the total number of Eq. is 1;
- Eq. (9) describes the equality of the pressure of the working fluid at the inlet PL and the sum of the pressures of all series-connected CP hydraulic tracts of arbitrary group (1 equation for PL and arbitrary j-th group CP); $i = 1..M_j$, where j is arbitrary in the range from 1 to N; the total number of Eq. is 1.

Eq. (10) describes the state of PL; the total number of Eq. is 1.

Thus, the generalized model of the ACPU system of a multi-unit PS with a sequential combination of CP hydraulic tracts consists $12 \cdot N_{PS} + 2$ equations.

The value of hydraulic inductivities and resistances in Tab. 1, calculated by expressions:

$$L'_{11_{i,j}} = \left(L_{12_{i,j}} L_{21_{i,j}} \left(L^2_{11_{i,j}} L''_{i,j} \right)^{-1} - L^{-1}_{11_{i,j}} \right)^{-1} \tag{11}$$

$$L'_{33_{i,j}} = \left(L_{32_{i,j}} L_{23_{i,j}} \left(L^2_{33_{i,j}} L''_{i,j} \right)^{-1} - L^{-1}_{33_{i,j}} \right)^{-1} \tag{12}$$

$$L'_{12_{i,j}} = L'_{21_{i,j}} = L_{11_{i,j}} L''_{i,j} L^{-1}_{12_{i,j}} \tag{13}$$

$$L'_{13_{i,j}} = L'_{31_{i,j}} = L_{11_{i,j}} L_{33_{i,j}} L''_{i,j} \left(L_{12_{i,j}} L_{23_{i,j}} \right)^{-1} \tag{14}$$

$$L'_{23_{i,j}} = L'_{32_{i,j}} = L_{33_{i,j}} L''_{i,j} L^{-1}_{23_{i,j}} \tag{15}$$

$$L'' = L_{12_{i,j}} L_{21_{i,j}} L^{-1}_{11_{i,j}} - L_{22_{i,j}} + L_{23_{i,j}} L_{32_{i,j}} L^{-1}_{33_{i,j}} \tag{16}$$

$$L_{11_{i,j}} = L_{tnom_{i,j}} + L_{\mu Hnom_{i,j}} + L_{\mu Qnom_{i,j}} \tag{17}$$

$$L_{12_{i,j}} = L_{21_{i,j}} = L_{\mu Qnom_{i,j}} \tag{18}$$

$$L_{23_{i,j}} = L_{32_{i,j}} = L_{\Delta Qnom_{i,j}} \tag{19}$$

$$L_{33_{i,j}} = L_{\Delta Qnom_{i,j}} + L_{\Delta Hnom_{i,j}} \tag{20}$$

$$L_{22_{i,j}} = L_{\mu Qnom_{i,j}} + L_{\Delta Qnom_{i,j}} \tag{21}$$

$$r_{11_{i,j}} = r_{12_{i,j}} = R_{\mu Qnom_{i,j}} \tag{22}$$

$$r_{22_{i,j}} = r_{23_{i,j}} = R_{\Delta Qnom_{i,j}} \tag{23}$$

$$r_{33_{i,j}} = R_{\Delta Qnom_{i,j}} + R_{\Delta Hnom_{i,j}} \tag{24}$$

where $L_{mnom_{i,j}}$, $L_{tnom_{i,j}}$, $L_{\mu Hnom_{i,j}}$, $L_{\mu Qnom_{i,j}}$, $L_{\Delta Qnom_{i,j}}$, $L_{\Delta Hnom_{i,j}}$, $R_{\mu Qnom_{i,j}}$, $R_{\Delta Qnom_{i,j}}$, $R_{\Delta Hnom_{i,j}}$ nominal parameters of the CP, which are calculated according to the method described in [11] by the geometric dimensions of its internal elements, taking into account the density and kinematic viscosity of the working fluid, which depend on its temperature. This makes it possible to take into account the effect of operational or emergency changes in pump parameters, physical properties, and temperature of the working fluid both on the pumping regimes and directly on the operation regimes of the electric drive directly during the simulation.

3. Results of test calculations

The correctness of the proposed mathematical model and the ability to work was verified by adapting for 4-unit ACPU ($N_{PS} = 4$, $N = 2$, $M_1 = 2$, $M_2 = 2$). $ACPU_{1,1}$ and $ACPU_{2,1}$ are the support aggregates, $ACPU_{2,1}$ and $ACPU_{2,2}$ are the main aggregates. Parameters of IM and CP of these aggregates are given in App. A. Some of the most important results of test

Tab. 1: The matrix $\mathbf{P}_{i,j}$ of ACPU parameters.

$\frac{r11_{i,j}}{L'_{11_{i,j}}} - \frac{r21_{i,j}}{L'_{12_{i,j}}}$	$\frac{r22_{i,j}}{L'_{12_{i,j}}} - \frac{r32_{i,j}}{L'_{13_{i,j}}}$	$\frac{r33_{i,j}}{L'_{13_{i,j}}} - \frac{r23_{i,j}}{L'_{12_{i,j}}}$	0	0	0	0	0
$\frac{r11_{i,j}}{L'_{21_{i,j}}} - \frac{r21_{i,j}}{L'_{22_{i,j}}}$	$\frac{r22_{i,j}}{L'_{22_{i,j}}} - \frac{r32_{i,j}}{L'_{23_{i,j}}}$	$\frac{r33_{i,j}}{L'_{23_{i,j}}} - \frac{r23_{i,j}}{L'_{22_{i,j}}}$	0	0	0	0	0
$\frac{r11_{i,j}}{L'_{31_{i,j}}} - \frac{r21_{i,j}}{L'_{32_{i,j}}}$	$\frac{r22_{i,j}}{L'_{32_{i,j}}} - \frac{r32_{i,j}}{L'_{33_{i,j}}}$	$\frac{r33_{i,j}}{L'_{33_{i,j}}} - \frac{r23_{i,j}}{L'_{32_{i,j}}}$	0	0	0	0	0
0	0	0	$\frac{-r_{m_{i,j}}\left(q_{CP_{i,j}}\right)}{L_{m_{i,j}}}$	0	0	0	0
0	0	0	0	$\frac{-R_{s_{i,j}}}{L_{s_{i,j}}} - j\omega_{s_{i,j}}$	0	$\frac{-1}{L_{s_{i,j}}}$	$\frac{-j\omega_{s_{i,j}}}{L_{s_{i,j}}}$
0	0	0	0	0	$\frac{-\omega_{s_{i,j}} R_{r_{i,j}}}{L_{r_{i,j}}} - j\left(\omega_{s_{i,j}} - \omega_{r_{i,j}}\right)$	$\frac{-\omega_{s_{i,j}}}{L_{r_{i,j}}}$	$\frac{j\left(\omega_{s_{i,j}} - \omega_{r_{i,j}}\right)}{L_{r_{i,j}}}$
0	0	0	0	0	0	1	0
0	0	0	0	1	1	0	$-R_{\mu_{i,j}}\left(\psi_{\delta_{i,j}}\right)$

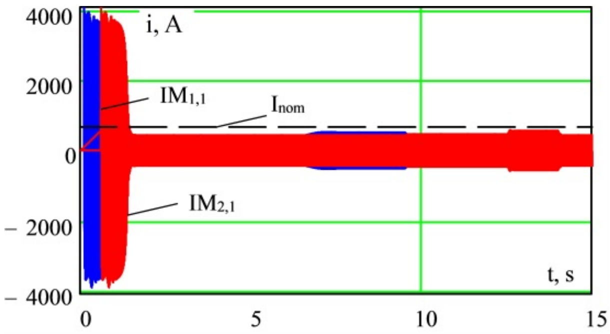
calculations using the developed mathematical model are presented in Fig. 2.

The total duration of the mathematical experiment was 15 seconds; the parameters of the equipment did not change during the simulation. The electric energy of the engines of the auxiliary installations was supplied manually for a time of 0.1 s ($ACPU_{1,1}$) and 0.6 s ($ACPU_{2,1}$), and the main ones – at the time of 1.5 s ($ACPU_{1,2}$) and 2.0 s ($ACPU_{2,2}$). After the stabilization of the aggregates mode, at the of time 9.5 s, the hydraulic connection with the pipeline of the sequentially connected support pump ($ACPU_{1,1}$) and the main pump ($ACPU_{1,2}$) took place, and at the of time 10.0 s - ($ACPU_{2,1}$) and ($ACPU_{2,2}$). After complete depressurization of the pipeline (at the time of 12.5 s), at the time of 13.5 s, all units were disconnected from it.

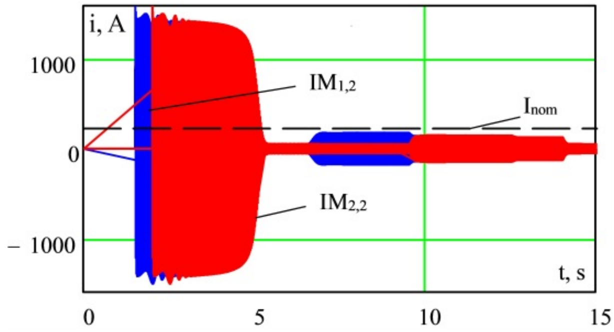
Fig. 2(a), 2(b), 2(c), and 2(d) shows the electromechanical coordinates, and Fig. 2(e), 2(f), and 2(g) show the hydraulic coordinates of the operating regimes of the 4-unit ACPU. For comparison with the results of the simulation, the same graphs show the nominal (catalog) values of the corresponding regime coordinates. The analysis of the transient processes occurring in the equipment, as well as the convergence of the coordinates of steady-state regimes with the passport parameters of IM and CP after the completion of these transient processes, give grounds for making a positive conclusion about the operability and correctness of the developed model.

4. Conclusion

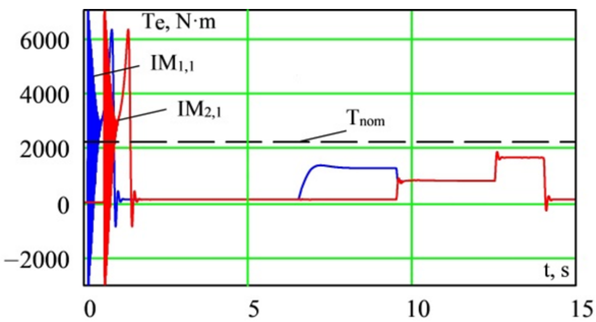
A mathematical model of a generalized multi-unit centrifugal pumping station with an asynchronous electric drive of pumps has been created. The model was verified by computer simulation of several operational and emergency regimes of operation of the system of four asynchronous centrifugal pump units. The use of the principle of electrohydraulic analogy made it possible to apply the basic principles of the theory of electric circuits to form the equations of the hydraulic subsystem. The achieved formalization makes it possible to automate the formation of equations to obtain an object model of a specific configuration. The use of pump parameters in the model, which depend the geometry of the internal elements, and on the physical properties of the liquid, makes it possible to study the impact of emergency and operational deviations of the equipment characteristics and operational changes in the physical properties of the working fluid on the operating regimes both at the level of individual modules and at the level of the pumping station as a whole. The model can be useful both at the design stage of new and for computer research of operating regimes of existing pumping stations and to improve the efficiency of its in order to avoid physical experiments. The created mathematical model of the established regimes of the generalized can be used as a basis for the development of specialized software as part of automated design systems. The created mathematical model can be applied for a complex analysis of electromagnetic, hydraulic, and thermal regimes, and electrotechnological efficiency of the electrotechnical complex of liquid pumping and heat network by combining it with relevant models. In the future, it is possible to significantly expand the functionality of the proposed model of the powerful equipment of the pumping station by integrating into it the physical model of its automatic control system. This will make it possible to obtain a highly effective tool for the study of such objects without conducting dangerous physical experiments on high-cost powerful equipment.



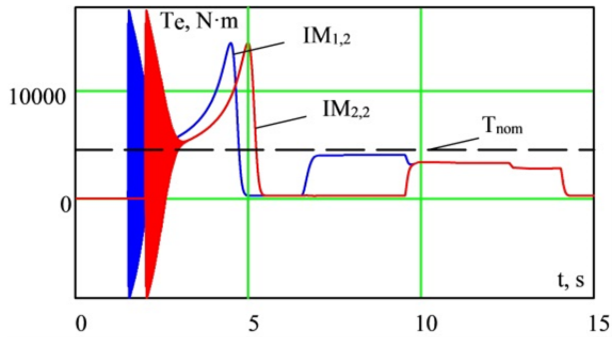
(a) Stator current of I of the supporting ACPU.



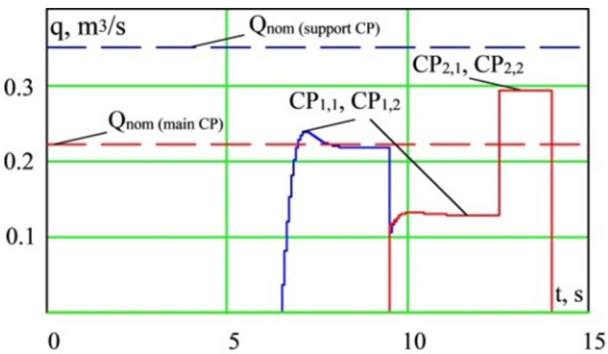
(b) Stator current of I of the main ACPU.



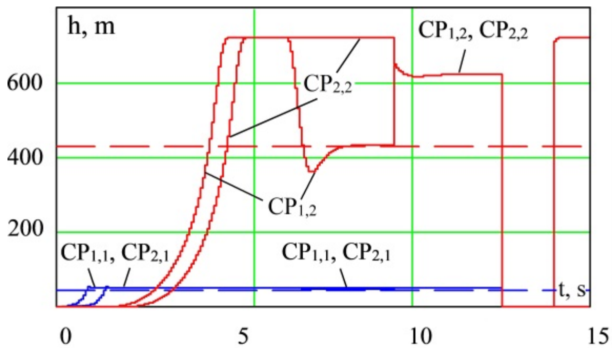
(c) Electromagnetic torque of I of the supporting ACPU.



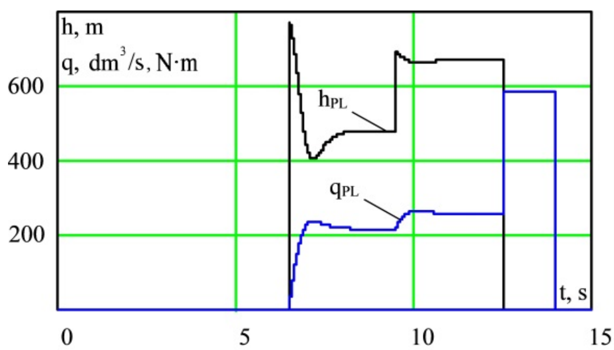
(d) Electromagnetic torque of I of the main ACPU.



(e) Volumetric flow rates of CP of the ACPU.



(f) Pressure of CP of the ACPU,



(g) Volumetric flow rates and pressure of the PS.

Fig. 2: Results of testing the mathematical model of 4-units PS.

References

[1] Verkhovna Rada of Ukraine. *Law Of Ukraine On Pipeline Transport*. The Official Bulletin of the Verkhovna Rada of Ukraine. Kyiv: Verkhovna Rada of Ukraine, 1996. 192/96-BP. [viewed date: 22 September 2022]. Available at: `https://zakon.rada.gov.ua/laws/show/en/192/96-ĐšÑĂ`

[2] OU K., H. RAO, Z. CAI, H. GUO, X. LIN, L. GUAN, T. MAGUIRE, B. WARKENTIN AND Y. CHEN. MMC-HVDC Simulation and Testing Based on Real-Time Digital Simulator and Physical Control System. *IEEE Journal of Emerging and Selected Topics in Power Electronics*. 2014, vol. 2, iss. 4, pp. 1109-1116. ISSN 2168-6777. DOI: 10.1109/JESTPE.2014.2337512

[3] JANEVSKA, G. Mathematical Modeling of Pump System. *In: Electronic International Interdisciplinary Conference*. Zilina : EDIS, 2013, vol. 2, pp. 455–458. ISBN: 9788055407623 8055407622.

[4] LOZYNSKYI, A., A. KUTSYK AND O. KINCHUR. The research of efficiency of the use of neuropredictor in the control system of water-supply pump electric drive. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*. 2017, iss. 1, pp. 93-99. ISSN 2071-2227.

[5] WANG. Y., H. ZHANG, Z. HAN AND X. NI. Optimization Design of Centrifugal Pump Flow Control System Based on Adaptive Control. *Processes*. 2021, 9(9):1538. ISSN 1407-7345. DOI: 10.3390/pr9091538.

[6] ALABIED. S., O. HAMOMD, A. DARAZ, F. GU AND A.D. BALL Fault diagnosis of centrifugal pumps based on the intrinsic time-scale decomposition of motor current signals. *In: 2017 23rd International Conference on Automation and Computing (ICAC)*. IEEE, 2017, pp. 1-6. ISBN 978-0-7017-0260-1. DOI: 10.23919/IConAC.2017.8082027

[7] GEVORKOV L., V. VODOVOZOV, T. LEHTLA, AND Z. RAUD. PLC-based hardware-in-the-loop simulator of a centrifugal pump. *In: 2015 IEEE 5th International Conference on Power Engineering, Energy and Electrical Drives (POWERENG): IEEE*, 2015, pp. 491-496. ISBN 978-1-4799-9978-1. DOI: 10.1109/PowerEng.2015.7266366.

[8] KOSTYSHYN, V., I. YAREMAK. Mathematical model of reliability and efficiency of pumping unit of an oil pumping station. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*. 2017, iss. 5, pp. 62-68. ISSN 2071-2227.

[9] KURLIAK, P. Analysis of energy flows in systems of different physical nature of electrically driven of turbomachines. *Technology audit and production reserves*. 2015, vol. 2, iss. 1(22), pp. 12-17. ISSN 2226-3780. DOI: 10.15587/2312-8372.2015.41406.

[10] PARANCHUK, Y., V. LYSIAK. Energy efficient power supply system and automatic control of modes of the "power supply - Pumping station" complex. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*. 2015, iss. 3, pp. 115-124. ISSN 2071-2227. DOI: 10.29202/nvngu/2018-3/16.

[11] KOSTYSHYN, V. Simulation regimes of centrifugal pumps based on electrohydraulic analogy. I*vano-Frankivsk: Ivano-Frankivsk National Technical University of Oil and Gas, 2000*. ISBN 966-7327-05-1.

[12] KOSTYSHYN, V., I. YAREMAK AND P. KURLIAK. Creation of object-oriented model of centrifugal pump on the basis of electrohydrodynamic analogy method. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*. 2019, iss. 6, pp. 72-79. ISSN 2071-2227. DOI: 10.29202/nvngu/2018-3/16.

[13] GOGOLYUK, P., T. GRECHYN, A. RAVLYK AND I. GRINBERG. Mathematical Modeling and Simulation of Transients in Power Distribution Systems with Valve Devices and Dynamic Loading. *In: 2003 IEEE Power Engineering Society General Meeting (IEEE Cat. No.03CH37491)*. IEEE, 2003, vol. 3, pp. 1580-1585. ISBN: 0-7803-7989-6. DOI: 10.1109/PES.2003.1267391.

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Appendix A Equipment Parameters

Tab. 2: Parameters of IM 4AN355M6U3 of the supporting ACPU.

$\mathbf{P_{nom}}$, kW	η_{nom}	$\mathbf{V_{s.nom}}$, V	$\mathbf{n_{nom}}$, rpm	$\cos(\varphi_{nom})$
250	0.935	380	985	0.9
$\mathbf{p_0}$	$\mathbf{T_{max*}}$	$\mathbf{T_{min*}}$	$\mathbf{T_{s*}}$	$\mathbf{I_{s*}}$
3	2,2	0,9	1,4	7

Tab. 3: Parameters of IM 4AZMV-1600/6000U2 of the main ACPU.

$\mathbf{P_{nom}}$, kW	η_{nom}	$\mathbf{V_{s.nom}}$, V	$\mathbf{n_{nom}}$, rpm	$\cos(\varphi_{nom})$
1600	0.961	6300	2979	0.9
$\mathbf{p_0}$	$\mathbf{T_{max*}}$	$\mathbf{T_{min*}}$	$\mathbf{T_{s*}}$	$\mathbf{I_{s*}}$
1	2.6	0.7	1.9	6

Tab. 4: Parameters of CP 4NDs-N of the supporting ACPU.

$\mathbf{H_{nom}}$, m	$\mathbf{Q_{nom}}$, $\mathbf{m^3 \cdot h^{-1}}$	η_{nom}, V	$\mathbf{n_{nom}}$, rpm	$\mathbf{P_{hydr.nom}}$, kW
45	1260	0.809	980	154
$\mathbf{H_{0.nom*}}$	$\mathbf{R_{\Delta Q*}}$	$\mathbf{L_{\Delta Q*}}$	$\mathbf{R_{\Delta H*}}$	$\mathbf{L_{\Delta H*}}$
1.302	29.47	9.49	$6.627 \cdot 10^{-4}$ 0.4144	
$\mathbf{L_{t*}}$	$\mathbf{L_{\mu H*}}$	$\mathbf{L_{\mu Q*}}$	$\mathbf{R_{mechH*}}$	$\mathbf{L_{mechH*}}$
0.00876	0.0352	0.2375	7.180	0.02287

Tab. 5: Parameters of CP 4QG300-2-100b of the main ACPU.

$\mathbf{H_{nom}}$, m	$\mathbf{Q_{nom}}$, $\mathbf{m^3 \cdot h^{-1}}$	η_{nom}, V	$\mathbf{n_{nom}}$, rpm	$\mathbf{P_{hydr.nom}}$, kW
428	800	0.745	2980	932
$\mathbf{H_{0.nom*}}$	$\mathbf{R_{\Delta Q*}}$	$\mathbf{L_{\Delta Q*}}$	$\mathbf{R_{\Delta H*}}$	$\mathbf{L_{\Delta H*}}$
2.641	43.89	15.12	$5.897 \cdot 10^{-5}$	0.4675
$\mathbf{L_{t*}}$	$\mathbf{L_{\mu H*}}$	$\mathbf{L_{\mu Q*}}$	$\mathbf{R_{mechH*}}$	$\mathbf{L_{mechH*}}$
1.03311	0.3122	2.3111	20.377	0.00436